因果关系

Cause and Effect

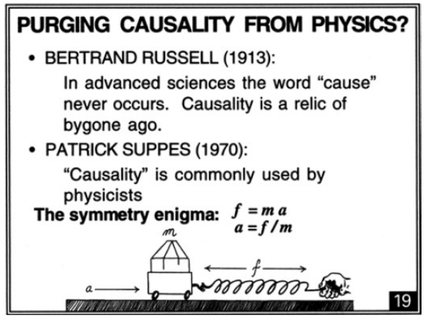
因果关系

- 概念
- 模型和简单推断

因果关系

- □概念
- □模型和简单推断

因果概念



- 事实和解释

  解释的变化：精灵、人、动物 -> 齿轮、杠杆、无生命物体 -> 粒子

- 因果在哪里存在？

  伽利略：先描述清楚，再解释（反对臆测，提倡实证）

  休谟：因果是观察的产物，是虚构的思维习惯

  Lord Russell 因果不对称但物理规律对称 f = ma（科学中没有因果，因果只是说话时方便）

  1888 Freancis Galton发现前臂和头大小很相关，其学生Pearson 提出相关非因果，在统计学中去除因果概念（contingency table）

  1913 Fisher制定随机试验用于从数据中检验因果关系，近几十年唯一的科学证明因果的方法，主流统计学唯一允许的因果概念

三种思维方式

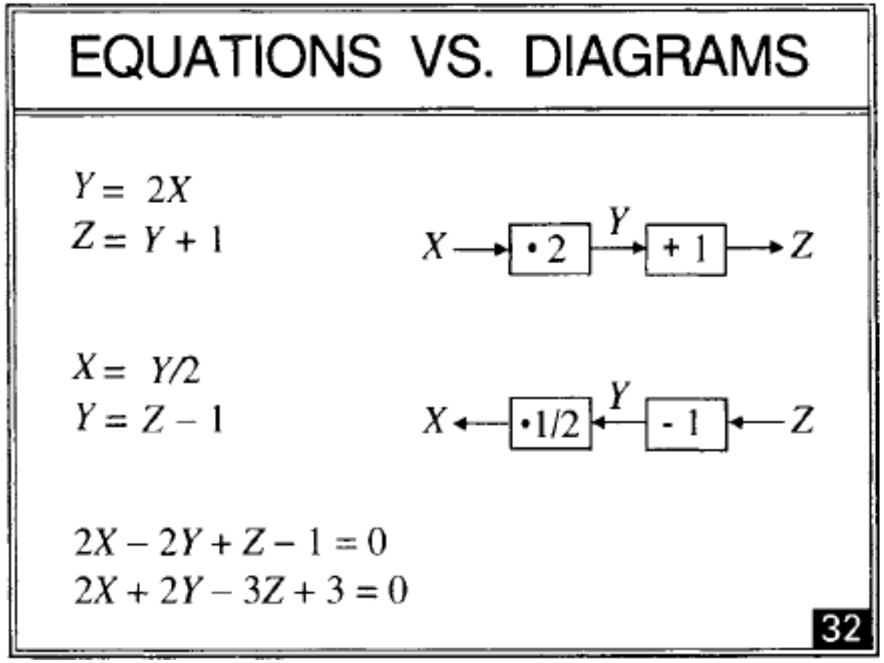
- Association
  - 三九感冒灵能治愈感冒
  - 鸡叫和太阳升起（周扒皮学鸡叫），蚂蚁搬家大雨哗哗，燕子低飞蛇过道大雨不久就来到
  - 太阳不下山和长生不老，此处祥云普照必有高僧
  - 这个人人品很差，道德败坏，作风恶劣，所以他说的不对
  - 人家都向权势者巴结攀附，然后地位提高了很多，获得很多实在的利益，你也可以这么办
- How to 和What if
  - 工程思维（发明创造、做工程、做成某事）
  - 只要做了A，就有可能B，反正B实现了，才不管是不是A引起的
- Why
  - 科学思维（相关非因果，目标不是做成某事，而是搞清楚某事为何被做成，以优化和改进其过程，使其大规模应用时受益）
  - 只要做了A1，一定条件下就几乎一定B，做了A2也可以做到B

统计学中因果稀少？科学中因果稀少？

- Weak Answer：
  - Association <- single uncontrolled experiment
  - Cause <- many controlled experiment

- Strong Answer：
  - No cause in the language of probability
  - No precision and computational benefits of a formal language (Galileo)

计算机中的因果？


EQUATIONS VS. DIAGRAMS
Y = 2X
Z = Y + 1
X → •2 → Y → +1 → Z
X = Y/2
Y = Z − 1
X ← •1/2 ← Y ← -1 ← Z
2X − 2Y + Z − 1 = 0
2X + 2Y − 3Z + 3 = 0
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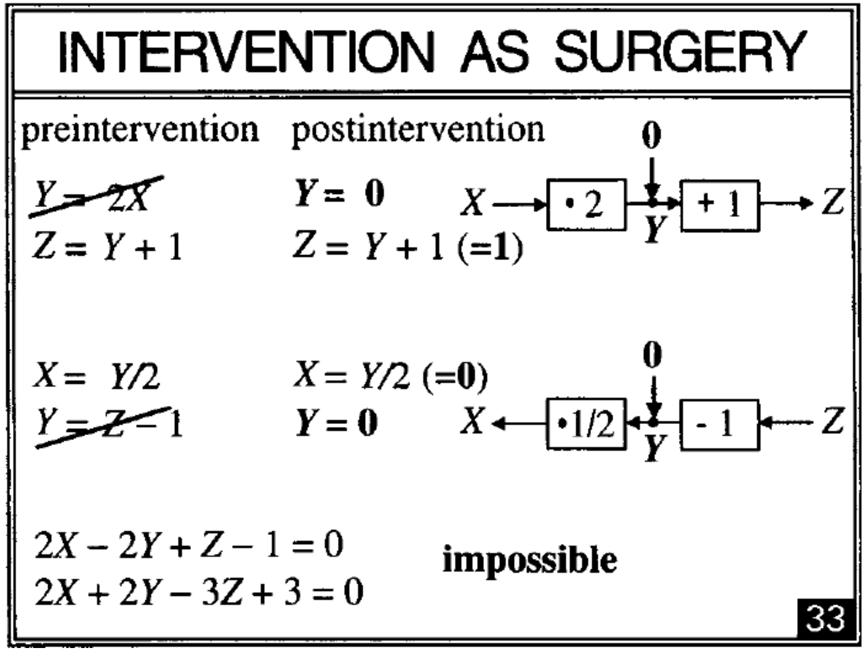
INTERVENTION AS SURGERY
preintervention
postintervention
Y = 2X
Z = Y + 1
Y = 0
Z = Y + 1 (=1)
X → •2 → Y → +1 → Z
0
X = Y/2
Y = Z − 1
X = Y/2 (=0)
Y = 0
X ← •1/2 ← Y ← -1 ← Z
0
2X − 2Y + Z − 1 = 0
2X + 2Y − 3Z + 3 = 0
impossible
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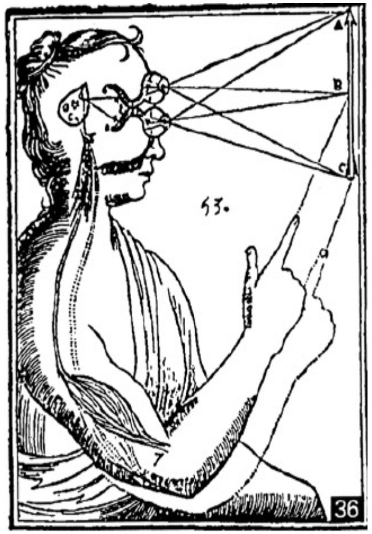
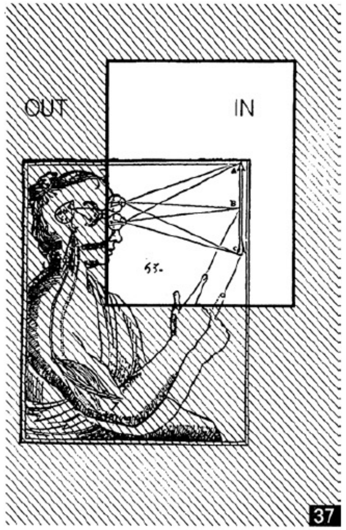
因果方向性的来源

- Boundary Condition
- Ins and Outs
- Manipulator and Outside intervention

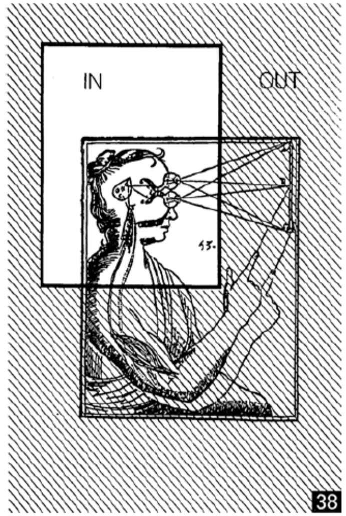
- 观察和干预即实验室



No cause and effect

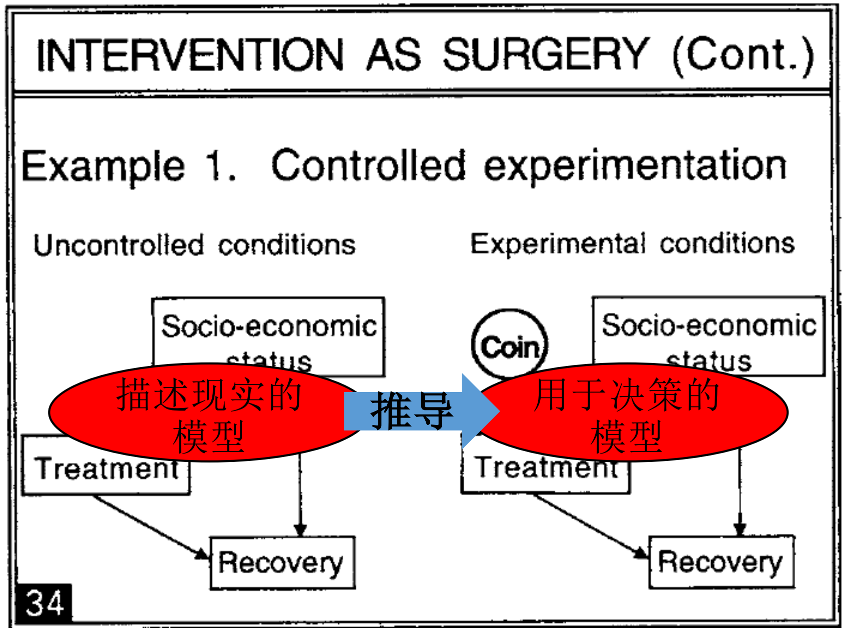


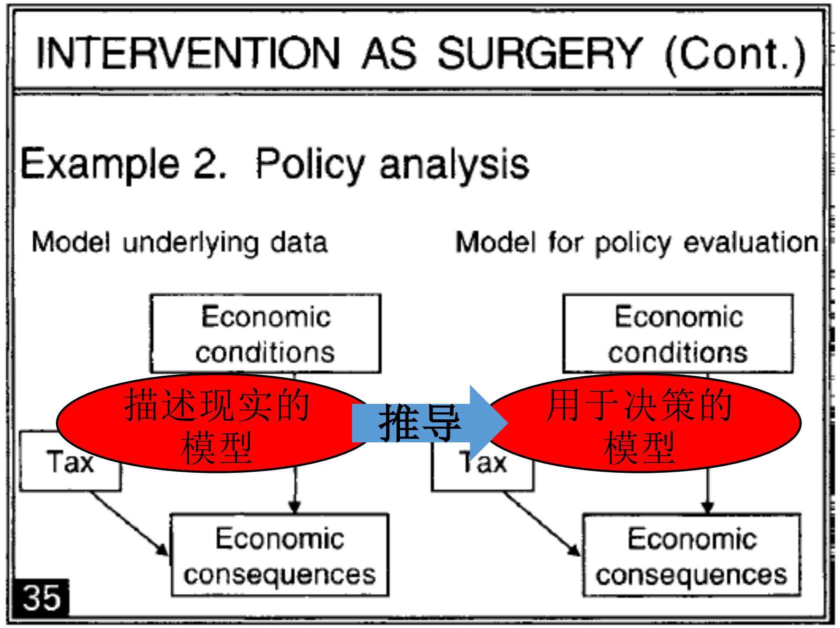
the motion of the hand *causing* this light ray to change angle



the light ray that causes the hand to move

干预的例子？

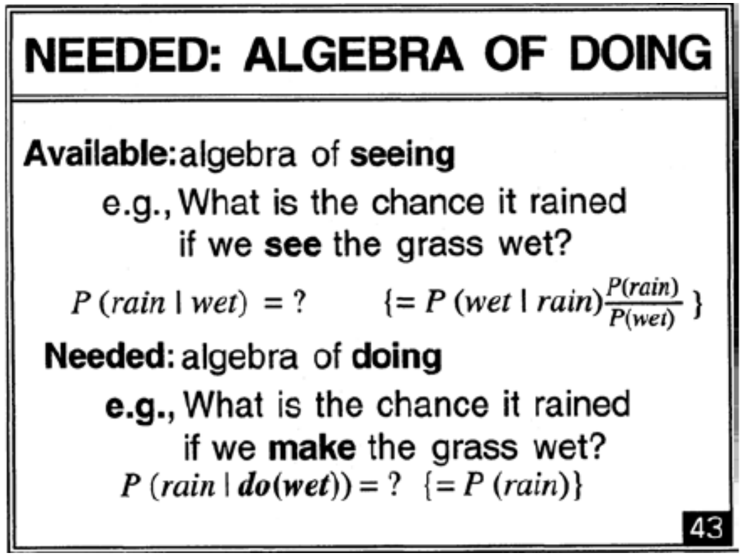




- Randomization and Intervention

如何描述干预？

- 代数、微积分、概率论等语言只描述了观察，不能很精确和可计算地描述干预。
- 可以使用一些符号来描述干预，比如用do，因为好写好记好读。



NEEDED: ALGEBRA OF DOING

Available: algebra of **seeing**

e.g., What is the chance it rained if we **see** the grass wet?

$P(rain \mid wet) = ?$ $\{= P(wet \mid rain)\frac{P(rain)}{P(wet)}\}$

Needed: algebra of **doing**

e.g., What is the chance it rained if we **make** the grass wet?

$P(rain \mid \mathbf{do}(\mathbf{wet})) = ?$ $\{= P(rain)\}$

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如何计算干预？

尝试提出一些干预化简规则

RULES OF CAUSAL CALCULUS

Rule 1: Ignoring observations

$$P(y \mid do\{x\}, z, w) = P(y \mid do\{x\}, w) \quad \text{if } (Y \perp\!\!\!\perp Z \mid X, W)_{G_{\overline{X}}}$$

Rule 2: Action/observation exchange

$$P(y \mid do\{x\}, do\{z\}, w) = P(y \mid do\{x\}, z, w) \quad \text{if } (Y \perp\!\!\!\perp Z \mid X, W)_{G_{\overline{X}\underline{Z}}}$$

Rule 3: Ignoring actions

$$P(y \mid do\{x\}, do\{z\}, w) = P(y \mid do\{x\}, w) \quad \text{if } (Y \perp\!\!\!\perp Z \mid X, W)_{G_{\overline{X}, \overline{Z(W)}}}$$

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Rule 1 可忽略与目标无关的观察

Rule 2认为干预导致的事实和非干预导致的事实对于观测者不应该有差别，即对于观测者，可用非干预产生的事实代替干预产生的事实（自由人或自由精神假设）

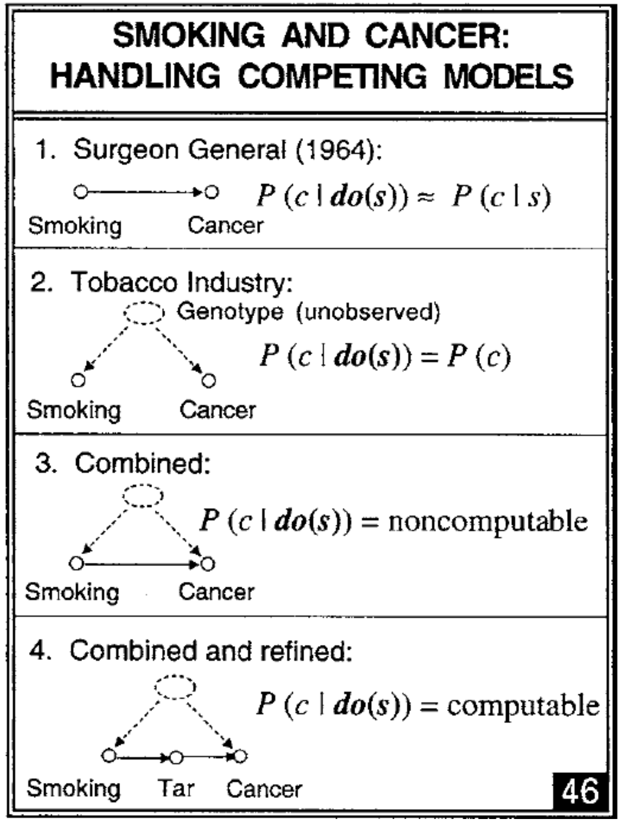
Rule 3 可忽略与目标无关的干预

干预的一个用处——吸烟导致肺癌？

吸烟导致肺癌？

- 1964年，Surgeon General发布了吸烟和肺癌的很强相关性报告，声称吸烟导致肺癌，禁烟就很少有人肺癌啦。我们禁烟吧！
- 烟草业声称有一种基因导致了对于尼古丁的渴望和肺癌，吸烟和肺癌没关系，都是基因惹的祸，不吸烟一样肺癌。不用禁烟啊！

吸烟导致肺癌？



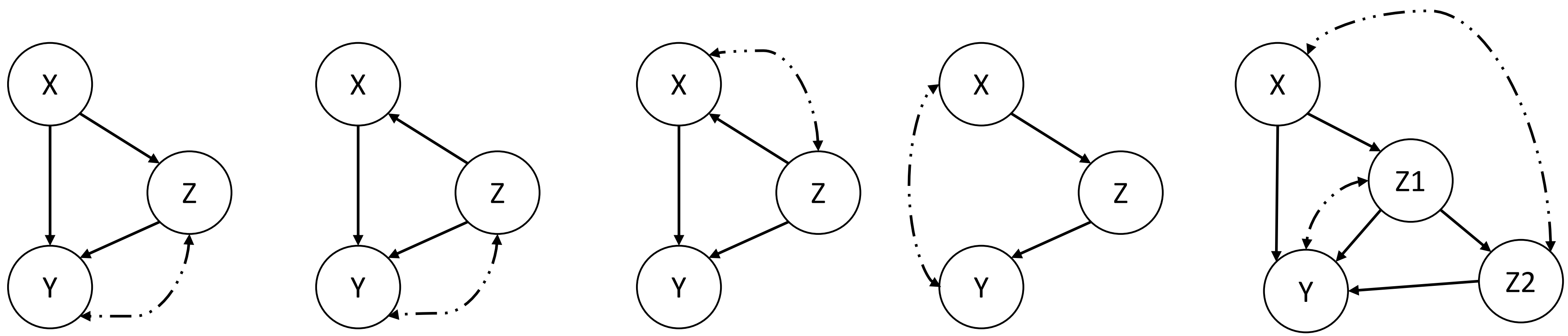
各方支持自己观点的模型

消除分歧的方法：烟草工业界承认存在一丁点因果关系，世界卫生组织承认存在一丁点遗传因素，大家使用合并后的模型；结果统计学家告诉他们从任何数据中都不可能评估出这个模型的连接关系，即不可计算，因为任何数据都可以同时符合模型1和2！

政治斗争继续……

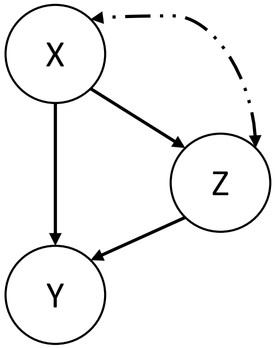
有人提出添加辅助因素tar in lung，再问统计学家，答案是使得模型可计算出闭式解了！

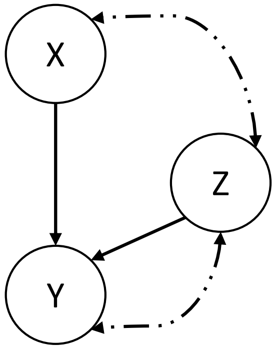
P(Y|do(X=x))可识别模型

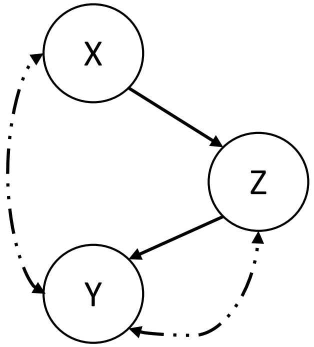


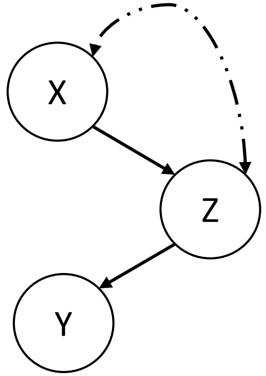
双向虚线箭头表示隐变量(不可观测但存在)对连接的两个变量都有影响

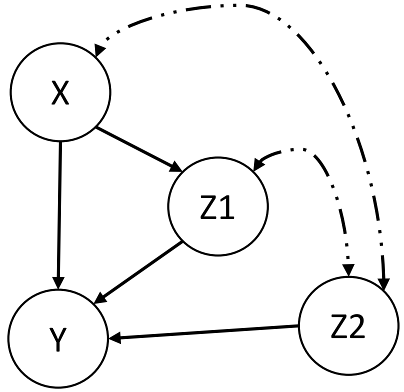
P(Y|do(X=x))不可识别模型

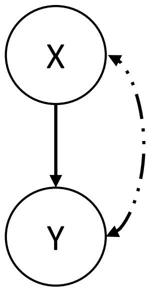


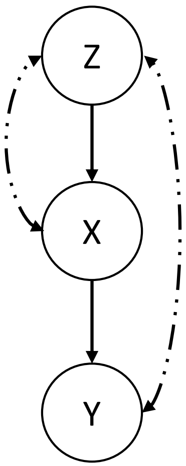












双向虚线箭头表示隐变量(不可观测但存在)对连接的两个变量都有影响

如何计算出P(Y|do(X=x))

Back-door criterion

do(X=x)

$$P(y \mid \hat{x}) = \sum_{z} P(y \mid x, z) P(z)$$

Z blocks all back trace from X to Y and Z isn’t descent of X

Front-door criterion

$$P(y \mid \hat{x}) = \sum_{z} P(z \mid x) \sum_{x'} P(y \mid x', z)\, P(x')$$

do(X=x)

blocks all directed trace from X to Z，(X->Z可识别)
X blocks every back trace from Z to Y，(Z->Y可识别)
Z intercepts all trace from X to Y，(X对Y影响仅通过Z)

要识别从X->Y的因果作用，我们不需要观测到所有的变量，只需要观测到切断后门路径或者前门路径的变量即可

前后门准则证明

Back-door Criterion

$$P(y \mid do(X) = x) = \sum_z P(y, z \mid do(X = x))$$
$$= \sum_z P(y \mid x, z)P(z). \#$$

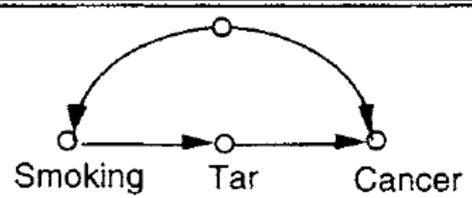
Front-door Criterion

$$Z \perp U | X, Y \perp X | (Z, U)$$

$$\begin{aligned}
&P(y \mid do(X) = x) \\
&= \sum_u P(y \mid x, u)P(u) \text{ (backdoor criterion of } U \text{ for } X \text{ and } Y) \\
&= \sum_u \sum_z P(y \mid x, z, u)P(z \mid x, u)P(u) \text{ (total probability)} \\
&= \sum_u \sum_z P(y \mid z, u)P(z \mid x)P(u) \text{ (independence)} \\
&= \sum_z P(z \mid x)P(y \mid do(Z) = z) \\
&\quad \text{(backdoor criterion of } U \text{ for } Z \text{ and } Y) \\
&= \sum_z P(z \mid x) \sum_{x'} P(y \mid x', z)P(x') \\
&\quad \text{(backdoor criterion of } X \text{ for } Z \text{ and } Y). \#
\end{aligned}$$

吸烟导致肺癌计算方法

TYPICAL DERIVATION IN CAUSAL CALCULUS



$P(c \mid do\{s\}) = \Sigma_t P(c \mid do\{s\}, t)\, P(t \mid do\{s\})$	Probability Axioms
$= \Sigma_t P(c \mid do\{s\}, do\{t\})\, P(t \mid do\{s\})$	Rule 2
$= \Sigma_t P(c \mid do\{s\}, do\{t\})\, P(t \mid s)$	Rule 2
$= \Sigma_t P(c \mid do\{t\})\, P(t \mid s)$	Rule 3
$= \Sigma_{s'} \Sigma_t P(c \mid do\{t\}, s')\, P(s' \mid do\{t\})\, P(t \mid s)$	Probability Axioms
$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\, P(s' \mid do\{t\})\, P(t \mid s)$	Rule 2
$= \Sigma_{s'} \Sigma_t P(c \mid t, s')\, P(s')\, P(t \mid s)$	Rule 3

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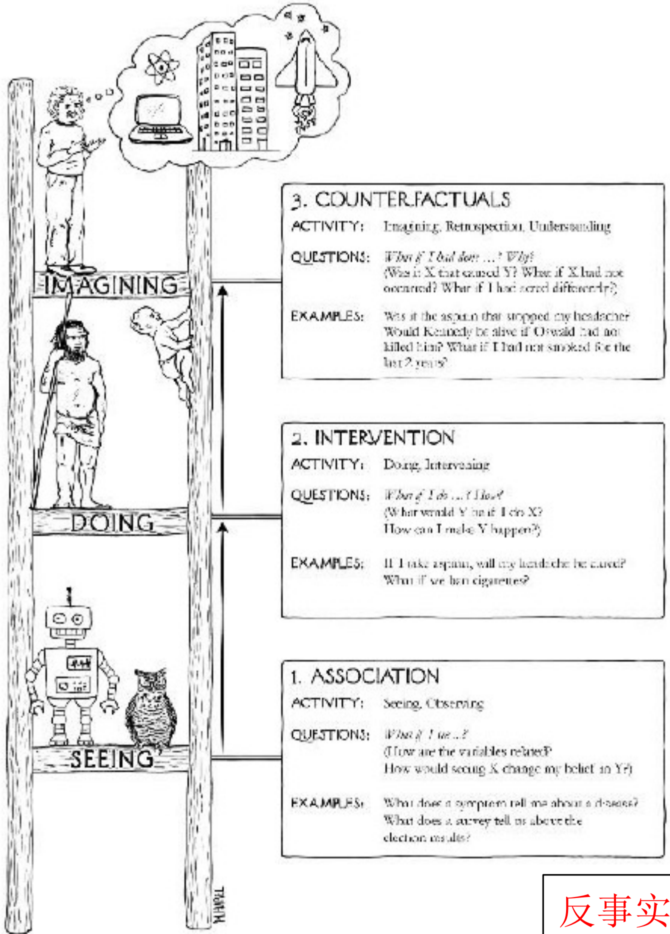
模型可计算了，但没有解决问题。主要问题是引入了两方可能不同意的假设，比如Surgeon General方认为吸烟和肺癌应该有不经过tar的直接连线。

另一个问题（总是有人说，你得考虑这些因素，加上这些节点）则可以很容易的用些测试（判断能否计算一个变量对另一个的影响）被统计学家看一眼（O(N)）解决。

因果关系

- 概念
- 模型和简单推断

因果之梯



Level (Symbol)	Typical Activity	Typical Questions	Examples
1. Association $P(y\|x)$	See...	What is?	What does a symptom tell me about a disease? What does a survey tell us about the election results?
2. Interventio... $P(y\|do(x), z)$			... I take aspirin, will ...dache be cured? ... we ban cigarettes?
3. Counterfactuals $P(y_x\|x', y')$	Imagining, Retrospection	Why? Was it X that caused Y?	Was it the aspirin that stopped my headache? ...d Kennedy be alive ...swald not shot him? ... if I had not been smoking the past 2 years?

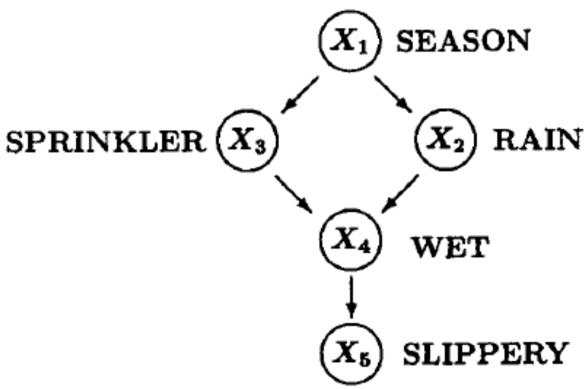
某些事实是个什么样子？应当如何表述它们？

我们看到了这个事实，那么如果我们这么做会怎么样呢?如何做才能发生我们想要的结果？

对于已发生的确定性的事实，当初我们如果那么做会怎么样，或者某些因素改变会怎么样？

反事实用处：法律上确定一个糟糕结果的责任归属（雨天车撞人问题、推理真凶问题）；确定历史人物对历史发展的影响；

用有向无环图G表示概率函数P



Definition : Markovian Parents

Let V = {*X*1,…, *Xn*} *be an ordered set of variables, and let P*(y) *be the joint probability distribution on these variables. A set of variables PAj is said to be* Markovian parents *of Xj if PAj is a minimal set of predecessors of Xj that renders Xj independent of all its other predecessors. In other words, PAj is any subset of* {*X*1,…, *Xj*-1} *satisfying*

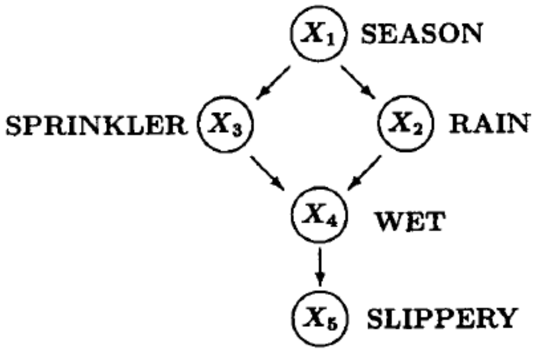
$$P(x_j \mid pa_j) = P(x_j \mid x_1, \ldots, x_{j-1}) \qquad (1.32)$$

and such that no proper subset of PAj satisfies (1.32).

用有向无环图G表示概率函数P

- **Definition : Markov Compatibility**
- *If a probability function P admits the factorization of* (1) *relative to DAG G, we say that G* represents *P, that G and P are* compatible*, or that P is* Markov relative *to G.*

$$P(x_1, \ldots, x_n) = \prod_i P(x_i \mid pa_i) \qquad (1)$$



用*d*-Separation来判断变量是否条件独立

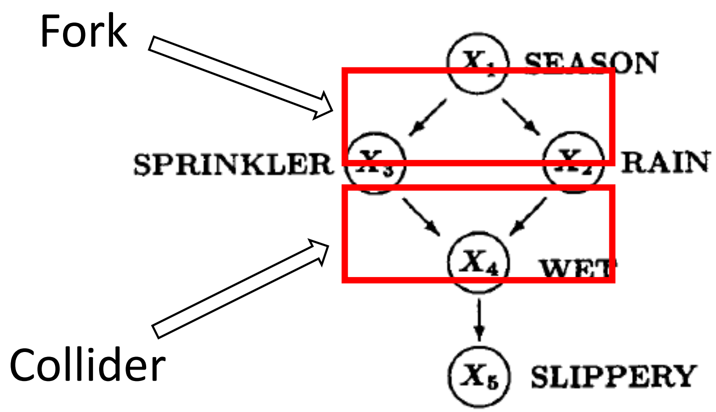
Definition : *d*-Separation

A path p is said to be d-separated (*or* blocked) *by a set of nodes Z if and only if*

1. *p contains a chain or a fork such that the middle node m is in Z, or*

2. *p contains an inverted fork* (*or* collider) *such that the middle node m is not in Z and such that no descendant of m is in Z.*

A set Z is said to d-separate X from Y if and only if Z blocks every path from a node in X to a node in Y.



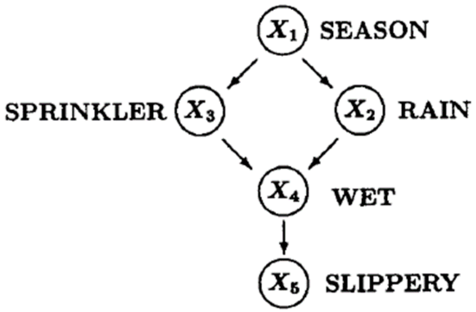
基于路径阻塞的d-separation

用*d*-Separation来判断变量是否条件独立

Theorem :Probabilistic Implications of *d*-Separation

If sets X and Y are d-separated by Z in a DAG G, then X is independent of Y conditional on Z in every distribution compatible with G. Conversely, if X and Y are not d-separated by Z in a DAG G, then X and Y are dependent conditional on Z in at least one distribution compatible with G.

向无环图G表示概率函数P时，X和Y被Z 有向分离等价于X和Y给定Z时条件独立。



因果模型中矛盾因素的选择

Simpon悖论

SIMPSON'S PARADOX

(Pearson et al. 1899; Yule 1903; Simpson 1951)

- Any statistical relationship between two variables may be **reversed** by including additional factors in the analysis.

Application: The adjustment problem

- Which factors **should** be included in the analysis.

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比如，发现吸烟的学生获得高分的多，但是根据年龄分组，却发现每个年龄组吸烟的学生获得低分的多；再添加父母收入因素，却发现每个年龄-收入组，吸烟的学生优势获得高分的多。
1975年UC-Berkeley入学男女入学率的性别偏见调查显示出同样的问题，总体体上看虽然男性较高，但按院系划分女性稍高。

Simpon悖论

SIMPSON'S PARADOX

(Pearson et al. 1899; Yule 1903; Simpson 1951)

- Any statistical relationship between two variables may be **reversed** by including additional factors in the analysis.

Application: The adjustment problem

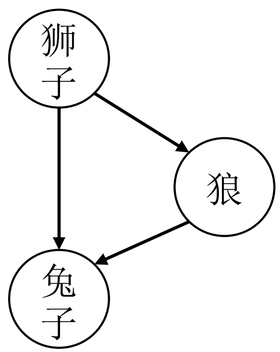
- Which factors **should** be included in the analysis.

48

比如，治疗对反应的影响，考虑性别、年龄、工资水平可测因素，遗传特性、生活方式等不可测因素。

比如，男性比同等学历的女性工资高（男性优势），但相同工资的男性却比女性学历高（女性优势）。

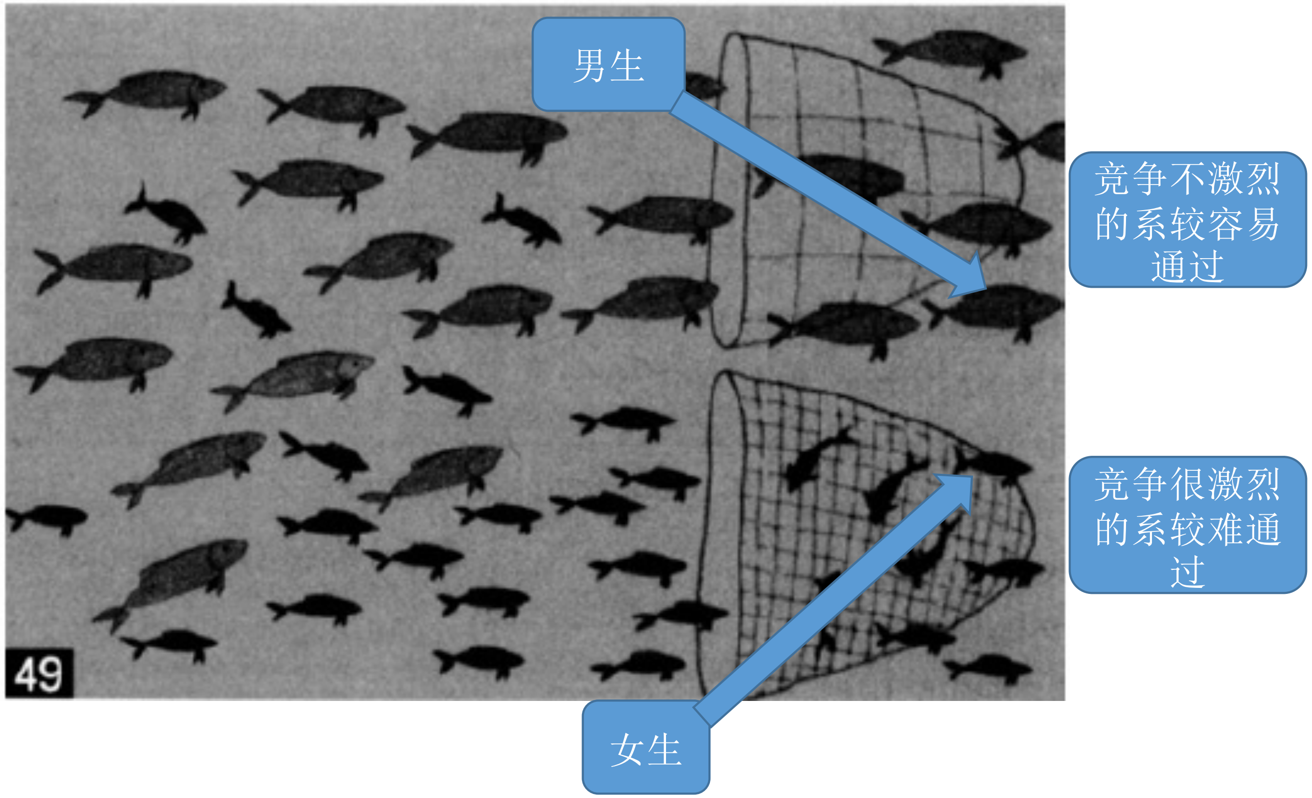
Simpon悖论



狮子数量增加是增加兔子的数量还是减少兔子的数量？

狮子增加会减少狼的数量进而增加兔子的数量，但狮子数量增加同时会减少兔子的数量。

矛盾因素产生的原因



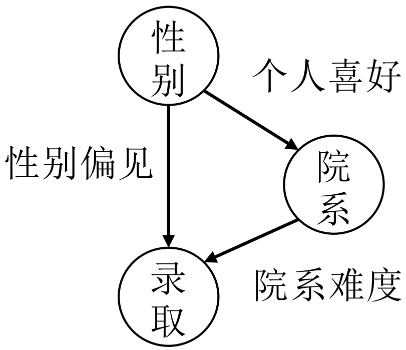
大鱼所男生的男生通过，小鱼所代表的女生没有通过，尽管小鱼事实上更容易通过。

因素X=性别

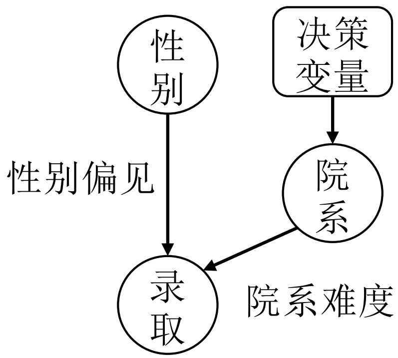
因素Y=录取

如何判断性别对录取的影响？

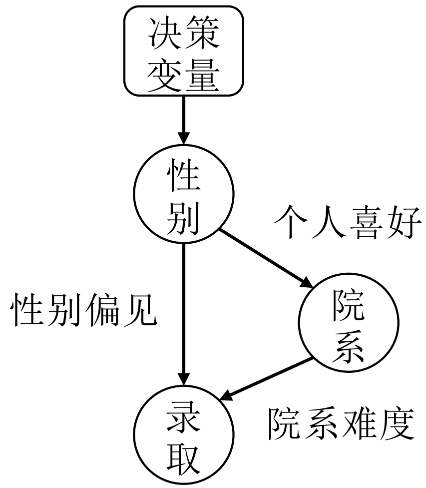
性别和院系对于录取结果的影响



导致辛普申悖论的因果网络



回答P(录取|do(改申EE)) 的
残缺网络(决策网络)

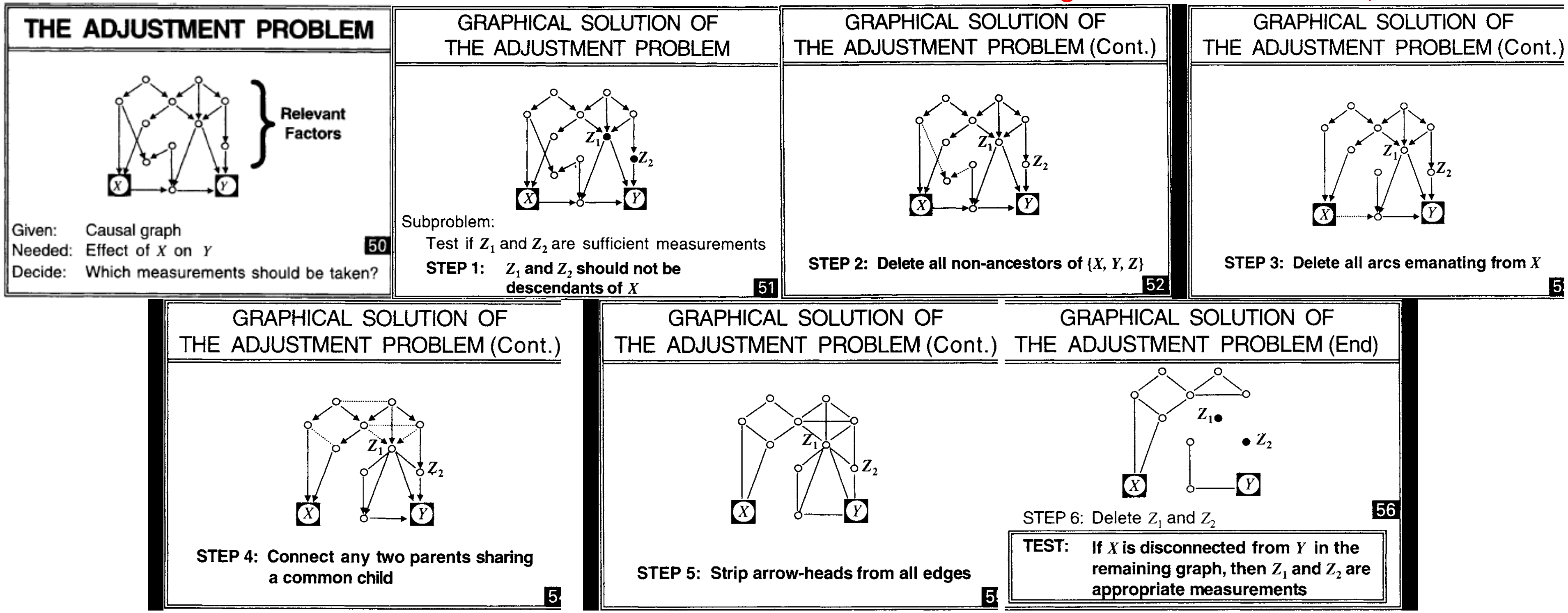


回答P(录取|do(变成女生)) 的
残缺网络(决策网络)

判断X->Y的影响需要考虑哪些因素？

Collider blocking

X is observed, and X blocks X^



基于道德图的d-separation

moralization

create base map

Create moral graph

d-separation

函数因果模型

- 节点间连线的机制由一个函数进行建模，这个函数的输入是外源性变量取值和前一个节点变量取值，输出是后一个节点变量取值。
- 函数是显示建模出来的描述父节点对子节点影响的内在机制，如特定的性别偏见（如男生0.7，女生0.3），个人偏好（如CS 0.6，EE 0.4），院系难度（CS 0.2 EE 0.8）。

总结

- 因果并不mysterious，也不metaphysical
- 没有讲如何从观测中学习(函数)因果模型，也没讲反事实概念在(函数)因果模型中具体应用
- 测试A->B的因果影响虽难（*d*-Separation概念可以有所帮助），但更难的是发现影响B的原因
- 还有很多问题等待解决，比如贫穷，癌症，不宽容，攀附，勤奋，幸运，效率等因素的因果关系。

参考资料

[1] Pearl J. Causality[M]. Cambridge university press, 2009.

[2] Pearl J, Mackenzie D. The book of why: the new science of cause and effect[M]. Basic Books, 2018.

[3] Koller D, Friedman N, Bach F. Probabilistic graphical models: principles and techniques[M]. MIT press, 2009.

[4] Pearl J. Theoretical impediments to machine learning with seven sparks from the causal revolution[J]. arXiv preprint arXiv:1801.04016, 2018.

[5] NIPS 17 – What If? Workshop Slides (PDF)（http://causality.cs.ucla.edu/blog/wp-content/uploads/2017/12/nips-dec2017-bw.pdf）

[6] NIPS 17 – What If? Workshop Slides (PPT [zipped])（hhttp://causality.cs.ucla.edu/blog/wp-content/uploads/2017/12/nips-dec2017-bw.pdf）

[7] http://causality.cs.ucla.edu/

本PPT大多数内容来自参考资料

参考资料及本PPT下载链接1: https://pan.baidu.com/s/1_bWK_CFPs7BSXeNDatQ1gQ 提取码: dwa7
参考资料及本PPT下载链接2: https://download.csdn.net/download/sikongpop/11156385