1 All identification result of 4 variable DAG with hidden confounders

The target variable is $x4$. The treatment variable is $x1$. The covariates are $x2$ and $x3$. The result was created by the program [1] and manually simplified further.

$$Not\ Identifiable \tag{1}$$

$$p(x4) \tag{2}$$

$$p(x4|x1) \tag{3}$$

$$p(x4|x1, x3) \tag{4}$$

$$p(x4|x1, x2) \tag{5}$$

$$p(x4|x1, x2, x3) \tag{6}$$

$$\sum_{x2} p(x4|x1, x2) \tag{7}$$

$$\sum_{X3} p(x4|x1, x3) \tag{8}$$

$$\sum_{X2} p(x2)p(x4|x1, x2) \tag{9}$$

$$\sum_{X3} p(x3)p(x4|x1, x3) \tag{10}$$

$$\sum_{x2} p(x2)p(x4|x1, x2, x3) \tag{11}$$

$$\sum_{X3} p(x3)p(x4|x1, x2, x3) \tag{12}$$

$$\sum_{x2,x3} p(x2, x3)p(x4|x1, x2, x3) \tag{13}$$

$$\sum_{X2,X3} p(x2|x1)p(x3|x1)p(x4|x1, x2, x3) \tag{14}$$

$$\sum_{X2,X3} p(x2)p(x3)p(x4|x1, x2, x3) \tag{15}$$

$$\sum_{X2,X3} p(x2,x3|x1) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{16}$$

$$\sum_{X2,X3} p(x2|x1)p(x3|x1) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{17}$$

$$\frac{\sum_{X2} p(x2)p(x1,x4|x2,x3)}{\sum_{X2} p(x2)p(x1|x2,x3)} \tag{18}$$

$$\frac{\sum_{X3} p(x3)p(x1,x4|x2,x3)}{\sum_{X3} p(x3)p(x1|x2,x3)} \tag{19}$$

$$\sum_{X3} p(x3) \frac{\sum_{x2} p(x2)p(x3,x4|x1,x2)}{\sum_{x2} p(x2)p(x3|x1,x2)} \tag{20}$$

$$\sum_{X2} p(x2) \frac{\sum_{x3} p(x3)p(x2,x4|x1,x3)}{\sum_{x3} p(x3)p(x2|x1,x3)} \tag{21}$$

$$\sum_{x2,x3} p(x3)p(x2|x1,x3) \sum_{x1,x3} p(x1,x3)p(x4|x1,x2,x3) \tag{22}$$

$$\sum_{x2,x3} p(x2)p(x3|x1,x2) \sum_{x1,x2} p(x1,x2)p(x4|x1,x2,x3) \tag{23}$$

$$\sum_{x1} p(x1) \sum_{x3} p(x4|x1,x3) \tag{24}$$

$$\sum_{X1} p(x1) \sum_{x2} p(x4|x1,x2) \tag{25}$$

$$\sum_{X2,X3} p(x3)p(x2|x1) \frac{\sum_{X1} p(x1)p(x3,x4|x1,x2)}{\sum_{X1} p(x1)p(x3|x1,x2)} \tag{26}$$

$$\sum_{X2,X3} p(x2)p(x3|x1) \frac{\sum_{X1} p(x1)p(x2,x4|x1,x3)}{\sum_{X1} p(x1)p(x2|x1,x3)} \tag{27}$$

$$\sum_{x3} p(x3|x1) \sum_{x1} p(x1)p(x4|x1,x3) \tag{28}$$

$$\sum_{x2} p(x2|x1) \sum_{x1} p(x1)p(x4|x1,x2) \tag{29}$$

$$\sum_{x2,x3} p(x3|x1,x2) \sum_{x1} p(x1,x2)p(x4|x1,x2,x3) \tag{30}$$

$$\sum_{x2,x3} p(x2|x1,x3) \sum_{x1} p(x1,x3)p(x4|x1,x2,x3) \tag{31}$$

$$\sum_{x2} p(x2|x1,x3) \sum_{x1,x3} p(x1,x3)p(x4|x1,x2,x3) \tag{32}$$

$$\sum_{x3} p(x3|x1,x2) \sum_{x1,x2} p(x1,x2)p(x4|x1,x2,x3) \tag{33}$$

$$\sum_{X3} p(x3|x2)p(x4|x1,x2,x3) \tag{34}$$

$$\sum_{X2} p(x2|x3)p(x4|x1,x2,x3) \tag{35}$$

$$\sum_{X3} \sum_{X2} p(x2)p(x3|x1,x2) \sum_{X1} p(x1|x2)p(x4|x1,x2,x3) \tag{36}$$

$$\sum_{X2} \sum_{X3} p(x3)p(x2|x1,x3) \sum_{X1} p(x1|x3)p(x4|x1,x2,x3) \tag{37}$$

$$\sum_{X2} p(x2|x1,x3) \sum_{X1} p(x1|x2)p(x4|x1,x2,x3) \tag{38}$$

$$\sum_{X3} p(x3|x1,x2) \sum_{X1} p(x1|x3)p(x4|x1,x2,x3) \tag{39}$$

$$\sum_{X3} p(x3|x1,x2) \sum_{X2} p(x2)p(x4|x1,x2,x3) \tag{40}$$

$$\sum_{X2} p(x2|x1,x3) \sum_{X3} p(x3)p(x4|x1,x2,x3) \tag{41}$$

$$\sum_{X2,X3} p(x3)p(x2|x1,x3) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{42}$$

$$\sum_{X2,X3} p(x2)p(x3|x1,x2) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{43}$$

$$\sum_{X2,X3} p(x2)p(x3|x1) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{44}$$

$$\sum_{X2,X3} p(x3)p(x2|x1) \sum_{X1} p(x1)p(x4|x1,x2,x3) \tag{45}$$

$$\sum_{X2,X3} p(x2|x1)p(x4|x1,x2,x3) \sum_{x1} p(x1)p(x3|x1,x2) \tag{46}$$

$$\sum_{X2,X3} p(x3|x1)p(x4|x1,x2,x3) \sum_{x1} p(x1)p(x2|x1,x3) \tag{47}$$

$$\sum_{x2,x3} p(x3|x1) \sum_{x1} p(x1,x2)p(x4|x1,x2,x3) \tag{48}$$

$$\sum_{X2,X3}\sum_{X1}\frac{\sum_{X3,X4}p(V)*p(V)}{\sum_{X4}p(V)}*\frac{\sum_{X2,X4}p(V)}{\sum_{X3}\sum_{X2,X4}p(V)} \tag{49}$$

$$\sum_{X2,X3}\frac{\sum_{X3,X4}p(V)}{\sum_{X2}\sum_{X3,X4}p(V)}*\sum_{X1}\frac{\sum_{X2,X3,X4}p(V)*p(V)}{\sum_{X3,X4}p(V)} \tag{50}$$

$$\sum_{X2,X3}p(x3)p(x2|x1)p(x4|x1,x2,x3) \tag{51}$$

2 All identification result of 3 variables DAG with hidden confounders

The target variable is Y. The treatment variable is T. The covariate is X. The result was created by the program [1] and manually simplified further.

$$Not\ Identifiable \tag{52}$$

$$p(Y|T,X)$$

$$p(Y|X)$$

$$\frac{p(X|Y,T)p(Y)}{\sum_Y p(X|Y,T)p(Y)}$$

$$\frac{\frac{p(V)}{\sum_X p(V)}*\sum_{X,T}p(V)}{\sum_Y \frac{p(V)}{\sum_X p(V)}*\sum_{X,T}p(V)}$$

$$\frac{\frac{\sum_{X,T}p(V)*p(V)}{\sum_X p(V)}}{\sum_Y \frac{\sum_{X,T}p(V)*p(V)}{\sum_X p(V)}}$$

3 Reference

[1] https://github.com/herdonyan/CausalIdentification